11') Método 1

$Ax - x = A^2$

$Ax - Ix = A^2$

Factorizo

$(A-I)x = A^2$

$(A-I)^{-1}.(A-I).x = (A-I)^{-1}.A^2$

$I.x = (A-I)^{-1}.A^2$

$x = (A-I)^{-1}.A^2$

$x = \begin{pmatrix} 1 & 0 \\ -1/2 & 1/2 \end{pmatrix} . \begin{pmatrix} 4 & 0 \\ 5 & 9 \end{pmatrix}$

$$\boxed{x = \begin{pmatrix} 4 & 0 \\ -1/2 & 9/2 \end{pmatrix}} \checkmark$$

Fila 1: $4+0=4$ $\quad 0+0=0$

Fila 2: $-2+2.5 = 1/2$ $\quad 0+9/2 = 9/2$

Sabemos que: $I.A = A$

y que: $A^{-1}.A = I$

$A = \begin{pmatrix} 1 & 0 \\ 1 & 3 \end{pmatrix}$

C.A

$A - I = \begin{pmatrix} 1 & 0 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$

$(A-I)^{-1}$:

1) $|(A-I)| \neq 0$

$1.2 - 1.0 = 2$ ✓

2) $\underset{(A-I)}{Adj} = \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}$

3) $\frac{\begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}}{2} = \begin{pmatrix} 1 & 0 \\ -1/2 & 1/2 \end{pmatrix} = (A-I)^{-1}$

$A^2 = A.A = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} . \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 5 & 9 \end{pmatrix}$

Fila 1: $4+0=4$ $\quad 0+0=0$

Fila 2: $2+3=5$ $\quad 0+9=9$

11) $AX - X = A^2$ $\quad A = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$ **MÉT. 2**

$I \cdot A = A$

$AX - IX = A^2$

$(A - I)X = A^2$

Definimos que:

$X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\downarrow$

$\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 5 & 9 \end{pmatrix}$

$\begin{pmatrix} a & b \\ a+2c & b+2d \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 5 & 9 \end{pmatrix}$ (*)

$\downarrow$

Fila 1:
$a \cdot 1 + c \cdot 0 = a$
$1 \cdot b + 0 \cdot d = b$

Fila 2:
$1a + 2 \cdot c = a + 2c$
$1 \cdot b + 2 \cdot d = b + 2d$

C.A

- $A - I = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$
- $A^2 = A \cdot A = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$

$A^2 = \begin{pmatrix} 4 & 0 \\ 5 & 9 \end{pmatrix}$

Fila 1:
$4 + 0 = 4$
$0 + 0 = 0$

Fila 2:
$2 + 3 = 5$
$0 + 9 = 9$

(*) Igualamos para obtener los elementos:

- $a = 4$
- $b = 0$
- $a + 2c = 5 \rightarrow 4 + 2c = 5 \rightarrow 2c = 5 - 4 \rightarrow 2c = 1 \rightarrow c = 1/2$
- $b + 2d = 9 \rightarrow 0 + 2d = 9 \rightarrow d = 9/2$

Entonces... $\boxed{X = \begin{pmatrix} 4 & 0 \\ 1/2 & 9/2 \end{pmatrix}}$ ✓